2014 MATHEMATICS OLYMPIAD

NORTHWEST MISSOURI STATE UNIVERSITY

SOPHOMORE TEST

DIRECTIONS

DO NOT OPEN this test booklet until instructed to do so.

Pull out the answer sheet from under the first page of the test booklet. With a NUMBER 2 LEAD PENCIL, to be used throughout the test, 1) print your name, 2) in the "Subject" blank fill in your grade: 10, 11 or 12, 3) Write your registration number in the nine blanks of the "I.D. Number" box on the right side of the answer card. Fill in the appropriate box for each number.

Read each question carefully, and for each question mark one answer only. Do not spend too much time on any one question. If a question seems to difficult, make a reasonable guess and continue to the next question. Your score will be the number of correct answers. If you change any answer, erase your original answer completely.

If you must leave the test room or if you need a new pencil or paper, etc., raise your hand to attract the attention of one of the proctors of the exam.

Calculators are **NOT** to be used during the exam.

PLEASE RETURN YOUR ANSWER SHEET,

REGISTRATION CARD

AND YOUR PENCIL.

YOU MAY KEEP YOUR TEST BOOKLET.

Sophomore Olympiad 2014

1. A car rents for \$290 per week plus \$0.10 per mile. The rental cost for a three-week trip of 600 miles is:

   A) \$350
   B) \$629
   C) \$930
   D) \$1829
   E) none of these

2. The least common multiple of 245 and 420 is:

   A) 35
   B) 210
   C) 2940
   D) 5880
   E) none of these

3. Where defined, $\frac{x^2-9}{3x^2-11x-4}\cdot\frac{2x^2-7x-15}{x^2+6x+9}\div\frac{2x^2-3x-9}{3x^2+10x+3}$ equals:

   A) $\frac{x-5}{x+4}$
   B) $\frac{x+5}{x-4}$
   C) $\frac{x-5}{x-4}$
   D) $\frac{x+5}{x+4}$
   E) none of these

4. If $a, b,$ and c are real numbers, $a \neq 0$, and the discriminant of $f(x)=ax^2+bx+c$ is negative, then the number of x-intercepts in the graph of the corresponding quadratic function is:

   A) 0
   B) 1
   C) 2
   D) cannot be determined
   E) none of these

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5. The sum of the solution(s) of $3x+\frac{4}{x-2}=\frac{-4x+12}{x-2}$ is:

A) $-\frac{4}{3}$

B) $-\frac{2}{3}$

C) $\frac{2}{3}$

D) $\frac{4}{3}$

E) none of these

6. An equation of the line perpendicular to $2x-7y=5$ and containing the point $(-2, 5)$ is:

A) $2x-7y=-39$

B) $2x-7y=31$

C) $7x+2y=-4$

D) $7x+2y=12$

E) none of these

7. In interval notation, the solution of $\left|\frac{1}{3}x-4\right|+3\geq 5$ is:

A) $[-6, 18]$

B) $(-\infty, 12]\cup[18, \infty)$

C) $\left(-\infty, \frac{2}{3}\right]\cup[2, \infty)$

D) $(-\infty, 6]\cup[18, \infty)$

E) none of these

8. The domain of the real valued function $f(x)=\sqrt{9-x}$ in interval notation is:

A) $(-\infty, 9]$

B) $[9, \infty)$

C) $(-\infty, -9)$

D) $(-3, 3)$

E) none of these

9. The vertex of the graph of the parabola $y = 2x^2 - 3x + 1$ is:

A) $\left(\frac{3}{4}, 1\right)$

B) $\left(\frac{3}{4}, -\frac{1}{8}\right)$

C) $\left(-\frac{3}{4}, \frac{35}{8}\right)$

D) $\left(\frac{1}{2}, 1\right)$

E) none of these

10. Three balls numbered 1 to 3 are placed in an urn, and three balls numbered 1 to 3 are placed in a second urn. If two balls are randomly selected, one from each urn, then the probability that the sum of the numbers on the balls is even is:

A) $\frac{1}{3}$

B) $\frac{5}{9}$

C) $\frac{1}{2}$

D) $\frac{5}{7}$

E) none of these

11. The number of vertices and edges on a pentagonal prism is:

A) 10 vertices and 15 edges
B) 15 vertices and 20 edges
C) 6 vertices and 10 edges
D) 25 vertices and 40 edges
E) none of these

12. The value of $[-5^2 + (6-8)^3 - (-4)] - [|-2|^3 + 1 - 3^2]$ is:

A) -37
B) -29
C) 21
D) 39
E) none of these

13. If $2x^3 - 3x^2y - 5xy^2$ is factored completely over the integers, then one factor is:

A) $2x$
B) $2x + 5y$
C) $x - 5y$
D) $2x - y$
E) none of these

14. In minipoker, each hand consists of 3 cards. If you are dealt 3 cards without replacement from a standard 52-card deck, then the number of ways to receive exactly 2 of a kind is:

A) 13
B) 26
C) 78
D) 104
E) none of these

15. The sum of the digits in a three-digit number is 18. The sum of the hundreds digit and the units digit is equal to the tens digit. If the hundreds digit and the units digit are interchanged, then the number is decreased by 99. The units digit of the original number is:

A) 1
B) 4
C) 5
D) 9
E) none of these

16. Where defined, $\frac{x+5}{x+1} + \frac{3}{1-x} - \frac{x^2-7}{x^2-1}$ is equal to:

A) $\frac{1}{x+1}$
B) $-\frac{1}{x+1}$
C) $\frac{x-9}{x^2-1}$
D) $\frac{x-15}{x^2-1}$
E) none of these

17. The coefficient of the x^2y^3 term in the expansion of $(x-2y)^5$ is:

A) -80
B) -50
C) 10
D) 80
E) none of these

18. Five runners, Andy, Beth, Caleb, Darnell, and Ella, are in a one-mile race. Andy finished the race seven seconds before Caleb. Caleb finished the race two seconds before Beth. Beth finished the race six seconds after Darnell. Ella finished the race eight seconds after Darnell. The order in which the runners finished the race is:

A) Andy, Darnell, Caleb, Beth, Ella
B) Andy, Ella, Caleb, Darnell, Beth
C) Caleb, Darnell, Beth, Andy, Ella
D) Andy, Beth, Caleb, Darnell, Ella
E) none of these

19. A scrabble player with seven different letters on his rack decides to test all possible five-letter arrangements before making his next play. If he tests one arrangement each second, then the number of minutes it will take before he is ready to play is:

A) 21
B) 42
C) 60
D) 84
E) none of these

20. A recipe calls for $1\frac{3}{5}$ cups of sugar for 18 brownies. The number of cups of sugar needed to make 12 brownies is:

A) 1
B) $1\frac{1}{15}$
C) $1\frac{1}{12}$
D) $1\frac{1}{5}$
E) none of these

21. The distance between (5, 7) and the center of the circle given by $4x^2+8x+4y^2-16y-16=0$ is:

A) $\sqrt{61}$
B) $\sqrt{97}$
C) 61
D) 97
E) none of these

22. The sum of the solution(s) of $\frac{3^{x^2}}{3^{3x}}=\frac{1}{9}$ is:

A) 0
B) 1
C) 2
D) 3
E) none of these

23. The area of the region determined by the system $y \geq |x|$ and $y \leq -|x+1|+4$ is:

A) $\frac{15}{4}$
B) $\frac{15}{2}$
C) 12
D) 15
E) none of these

24. In square $ABCD$ with side length s, the midpoints of $\overline{AD}$ and $\overline{DC}$ are connected to each other and to B to form a triangle. The area of the triangle in terms of s is:

A) $\frac{1}{2}s^2$
B) $\frac{1}{3}s^2$
C) $\frac{3}{8}s^2$
D) $\frac{3}{4}s^2$
E) none of these

25. If $f(x)=\begin{cases} x^2 & \text{for } x \le 1 \\ 1 & \text{for } x > 1 \end{cases}$ and $g(x)=\begin{cases} 2x & \text{for } x \le -1 \\ 3x & \text{for } x > -1 \end{cases}$, then $f(g(f(-2)))$ is:

A) -64
B) 1
C) 12
D) 144
E) none of these

26. A traveler spends 6 hours walking along a level road, up a hill, and returning home along the same route. If his pace is a constant 4 miles per hour on the level road, a constant 3 miles per hour uphill, and a constant 6 miles per hour downhill, then the total number of miles he walked is:

A) 20
B) 24
C) 26
D) 30
E) none of these

27. The Mathlete team is exactly 37.5% female. If no additional males join the team, then the smallest number of females who can join the team to raise the percentage to exactly 45% is:

A) 4
B) 8
C) 12
D) 15
E) none of these

28. The principal square root of the sum of the squares of the positive solution(s) of

$(9x^2-4)^4-10(9x^2-4)^2+9=0$ is:

A) 0
B) 1
C) $\frac{4}{3}$
D) $\frac{16}{9}$
E) none of these

29. The number that is half-way between $-\frac{3}{4}$ and $-\frac{5}{6}$ is:

A) $-\frac{4}{5}$

B) $-\frac{5}{9}$

C) $-\frac{4}{9}$

D) $-\frac{5}{12}$

E) none of these

30. The value of the expression $\frac{\sqrt{2}+\sqrt{10}}{2\sqrt{3+\sqrt{5}}}$ is equal to:

A) 1

B) $\sqrt{2}$

C) $\sqrt{5}$

D) $2\sqrt{3}$

E) none of these

31. The sum of the real solution(s) of $\frac{(x^2-13x+40)(x^2-13x+42)}{\sqrt{x^2-12x+35}}=0$ is:

A) 14

B) 19

C) 26

D) 38

E) none of these

32. On Halloween morning, a store owner decided to sell her last two costumes for $18 each. She made a profit of 25% on one costume and took a 20% loss on the other costume. Her total profit or loss for these two sales is a:

A) $2.50 loss

B) $0.90 loss

C) $0.50 profit

D) $5.00 profit

E) none of these

33. A circle is inscribed in a right triangle with point P common to both the circle and the hypotenuse AB. If $AP \cdot PB = 24$, then the area of the triangle is:

A) 6
B) 12
C) $10\sqrt{2}$
D) $6\sqrt{3}$
E) none of these

34. The first two terms of a sequence are $a_1 = 4$ and $a_2 = 2$. Each subsequent term is defined by the rule $a_n = a_{n-1} - a_{n-2}$. The value of a_{2014} is:

A) -4
B) -2
C) 2
D) 4
E) none of these

35. In interval notation, the solution set of $x^2 + 6x + 9 < 0$ is:

A) $(-3, 3)$
B) $(-\infty, -3) \cup (3, \infty)$
C) $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
D) $(-\infty, \infty)$
E) none of these

36. Ms. Thomas just opened her new restaurant in Maryville. Customers can choose from 1 of 4 appetizers, 1 of 5 entrées, and any 2 of 5 vegetables as side items with their meals. The number of different orders of 1 appetizer, 1 entrée and 2 vegetables that is possible for a customers to choose is:

A) 11
B) 40
C) 100
D) 200
E) none of these

37. The slope of a line parallel to the line passing through the points $(-3,-1)$ and $(-3,4)$ is:

A) $-\frac{5}{6}$
B) 0
C) $\frac{6}{5}$
D) undefined
E) none of these

38. Given the following premises, which conclusion is valid:

All humans are warm-blooded.
No reptiles are human.

A) Therefore, no reptiles are warm-blooded.
B) Therefore, all warm-blooded animals are human.
C) Therefore, there exists a human that is a reptile.
D) All reptiles are warm-blooded.
E) none of these

39. Jaime had a collection of football cards. He gave half of his collection his brother, Spencer. He then gave a fourth of what he had left to his sister, Tracy. That left him with 24 football cards. The number of football cards he had before he gave some to his siblings was:

A) 32
B) 40
C) 64
D) 192
E) none of these

40. Triangle *ABC* has vertices $A=(3,0)$, $B=(0,3)$, and *C*, where *C* is on the line $x+y=7$. The area of triangle *ABC* is:

A) 6
B) 8
C) 12
D) cannot be uniquely determined
E) none of these

41. If a circle is centered at O, $\overline{AB}$ is a diameter, and C is a point on the circle such that the measure of $\angle COB$ is 50°, then the measure of $\angle CAB$ in degrees is:

A) 20
B) 25
C) 45
D) 50
E) none of these

42. If $f(x)=ax^2+bx+c$ and $f(x+3)=3x^2+7x+4$, then $a+b+c$ is:

A) -1
B) 0
C) 1
D) 2
E) none of these

43. The U.S. Weather Bureau reported the following high temperatures for 10 major cities in the United States: 75, 65, 50, 90, 80, 60, 70, 80, 80, 85. The median for this data set is:

A) 70
B) 73.5
C) 77.5
D) 80
E) none of these

44. A month with 31 days has the same number of Mondays and Wednesdays. Out of the seven days of the week, the number of days that could be the first day of this month is:

A) 2
B) 3
C) 4
D) 5
E) none of these

45. The sum of the solutions of $x=\left|2x-\left|60-2x\right|\right|$ is:

A) 32
B) 60
C) 92
D) 120
E) none of these

46. If n is a positive integer, then a number that is not a perfect square is:

A) 2^{2n}

B) 4^{n}

C) 5^{6n+2}

D) 3^{12n}

E) none of these

47. The sum of all the integers between 50 and 350 which end in 1 is:

A) 4566

B) 4877

C) 5208

D) 5539

E) none of these

48. The remainder when $x^5 - 4x^3 - 3x + 1$ is divided by $x + 2$ is:

A) -17

B) -13

C) -5

D) 7

E) none of these

49. The value of $\dfrac{\left(5^{2/3}\right)\left(25^{1/3}\right)\left(-8^{1/3}\right)\left(2^{1/3}\right)}{10^{1/3}}$ is:

A) -10

B) -5

C) $\left(5^{1/3}\right)\left(2^{4/3}\right)$

D) $(-2)\left(25^{2/3}\right)$

E) none of these

50. Two angles are supplementary. The measure of the larger angle is 15° more than twice the measure of the smaller angle. The measure of an angle complementary to the smaller angle in degrees is:

A) 35

B) 45

C) 55

D) 75

E) none of these